Hydraulics theory

- ***Force***
- ***Pressure***
- ***Hydrostatics and Hydraulics***
- ***Example: hydraulic press***
- ***A simple hydraulic system***

Hydraulics:

"The science related to the flow of fluids"

Force

Definition

To **move** a body of a certain **mass** (m), a certain **force** (F) is needed.

The relationship between force and mass varies directly with acceleration (a).

Formula: $F = m \times a$

Units: $N = 1\ kg \times 1\ m/s^2$ (Newton)

1 Newton is the force needed to move a mass of 1 kg at 1 m/s in 1s.

Notes

“Kilogramme force” (kgf) is a unit which is still used in practice, but not recognised any longer.

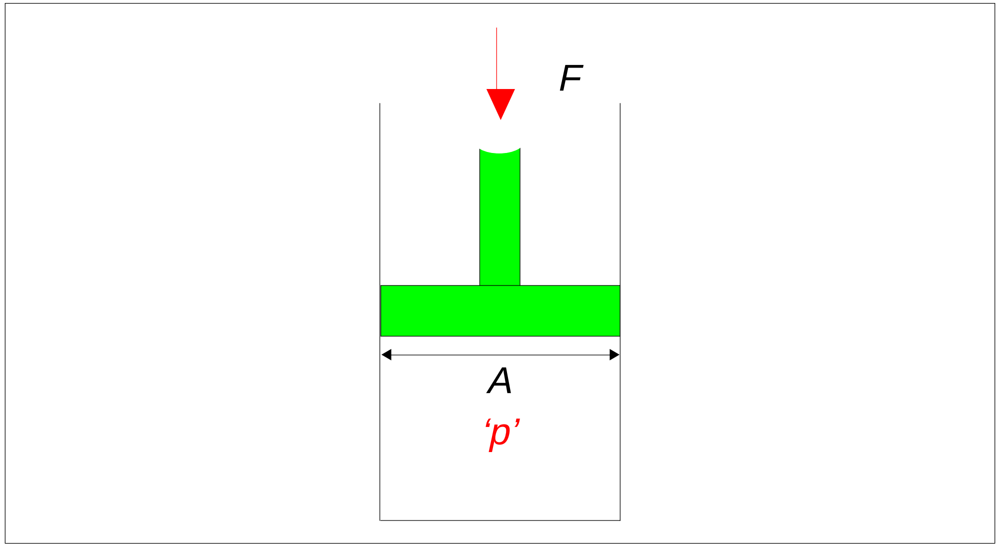
To convert from “kgf” to “N” multiply by 9,81.

1 kgf = 9.81 N

Pressure

Definition

When a force (F) acts on an enclosed fluid of surface area (A), a pressure (p) is generated.



Formula: $p = F / A$

Units: $1\ Pa = 1\ N / 1\ m^2$

1 Pascal is a pressure of 1 Newton on a surface of $1m^2$.

Notes

1 MPa (Megapascal) $= 10^6$ Pa
$= 10^6\ N/m^2$
$= 1N / mm^2$

$1\ kg / cm^2 = 0.0981$ MPa » 0.1 MPa » 1 bar

1 bar $= 10^5$ Pa = 14.5 psi (pounds / $inch^2$)

Hydrostatics and hydraulics

Principles

1. Liquids are relatively incompressible.

2. Pressure in a liquid can be generated by

- the mass of the liquid,
- an external force (mechanical pressure),
- the fluid velocity.

3. Pascal's Law: "Pressure applied on a confined fluid is transmitted undiminished in all directions, and acts with equal force on equal areas, and acts at right angles to them".

$$p_0 = F_0 / A_0$$

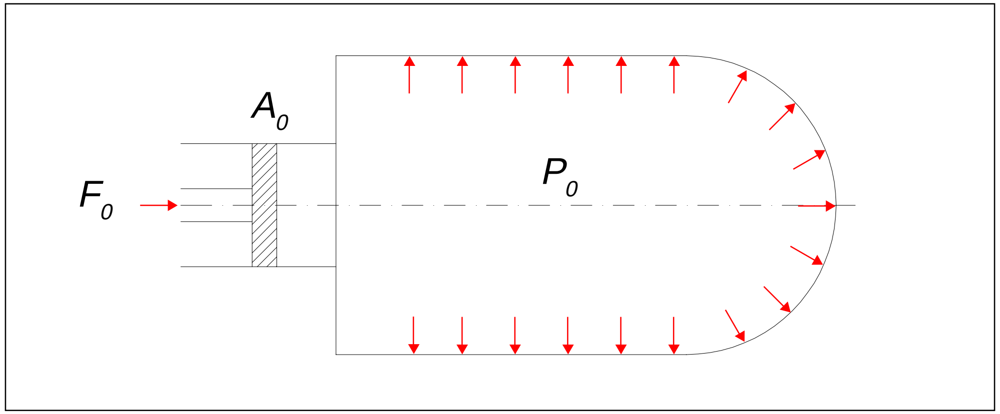
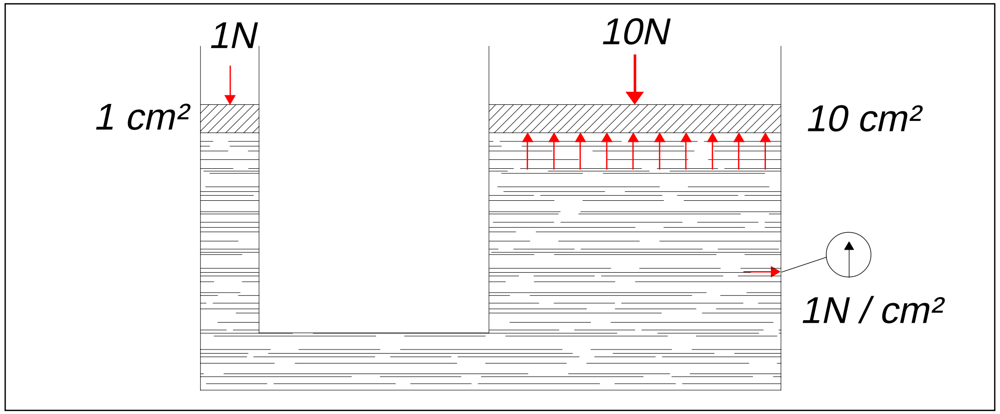


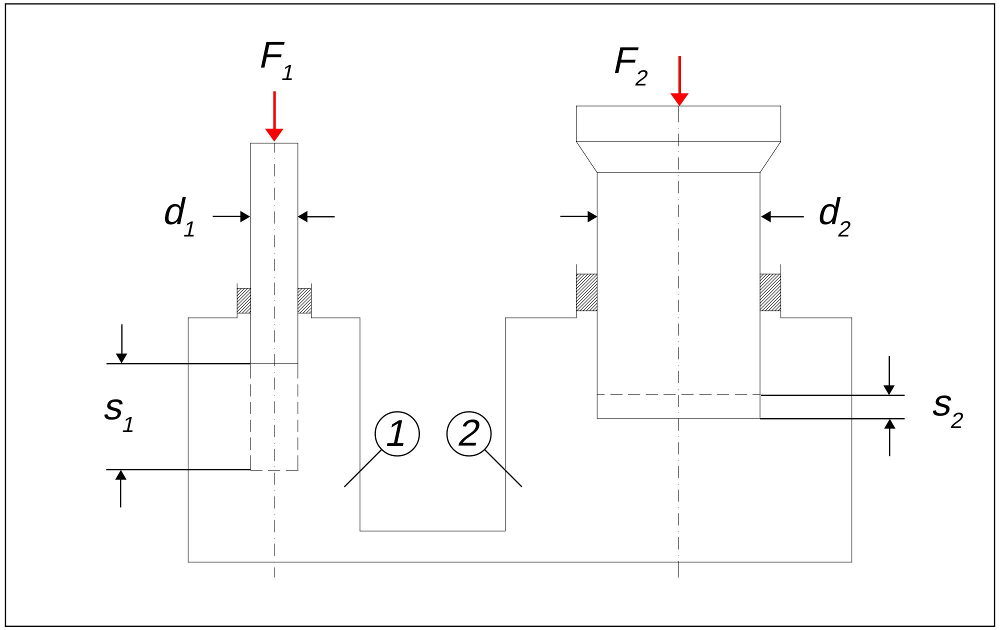
Illustration:
two interconnected cylinders filled with oil

When a force is applied on a surface area of a liquid, pressure will be the same in every point of that liquid.
A surface ten times as big will experience ten times the force.

To achieve equilibrium in this system, the big piston will need to be loaded with a force of 10N.



Example: hydraulic press



The principle of pressure propagation is used in excavators, presses, brakes,... where relatively small pressures are transformed into much greater forces.

Relation between forces and diameter

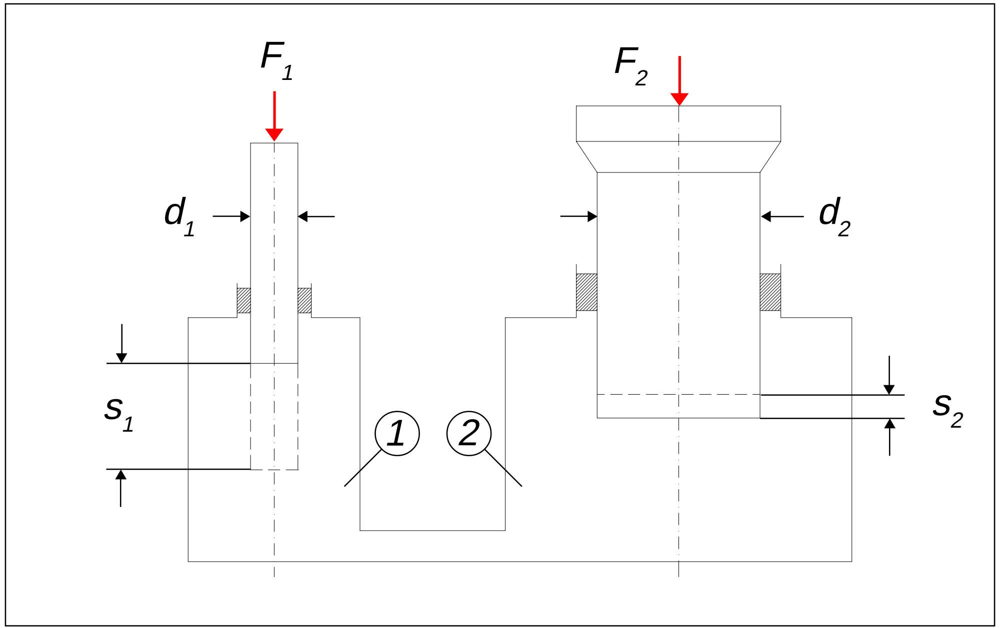
The two pistons have diameters d_1 and d_2, and therefore a surface A_1 and A_2
(area of a circle is pr^2 or $pd^2 / 4$)
If a force F_1 is applied on small piston, pressure generated is:
$p = F_1 / A_1$

Therefore:
$F_1 = p \times A_1$
$= p \times pd_1^2 / 4$

To achieve equilibrium
$p = F_2 / A_2$
$F_2 = p \times A_2$
$= p \times pd_2^2 / 4$

Therefore:
$F_1 / F_2 = (p \times pd_1^2 / 4) / (p \times pd_2^2 / 4)$
$F_1 / F_2 = d_1^2 / d_2^2$

<u>The forces on the pistons are proportional to the squares of the diameters</u>



Fluid displacement

When the first piston is moved, a volume of fluid is displaced:

$V_1 = A_1 \text{ x } s_1$

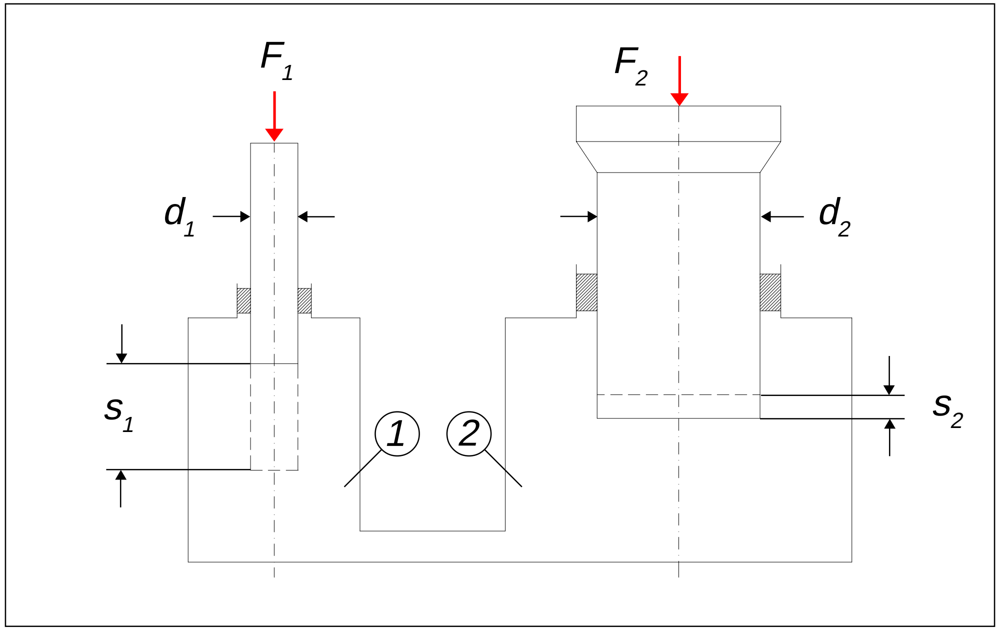
For the second piston, volume displaced is:

$V_2 = A_2 \text{ x } s_2$

Given the incompressibility of fluids:

$$V_1 = V_2$$
$$A_1 \text{ x } s_1 = A_2 \text{ x } s_2$$
$$(pd_1^2 / 4) \text{ x } s_1 = (pd_2^2 / 4) \text{ x } s_2$$
$$d_1^2 \text{ x } s_1 = d_2^2 \text{ x } s_2$$
$$s_1 / s_2 = d_2^2 / d_1^2$$

<u>The displacements are inversely proportional to the squares of the diameters</u>



Calculation

Assume:

A_1 = 50 cm²
A_2 = 100 cm²
F_2 = 1000 N
s_2 = 40 cm (distance a car that has to be lifted)

If the valve is shut, pressure in (2) will be:

$p_2 = F_2 / A_2$ = 1000 N / 100 cm² = 1 bar

To raise car s_2 (cm), volume (V) displaced by piston 2 will be:

V = s_2 x A_2 = 40 cm x 100 cm² = 4000 cm³ = 4 l

Stroke of piston 1 (s_1) needs to be:

s_1 = V / A_1 = 4000 cm³ / 50 cm² = 80 cm

Pressure needed to raise load:

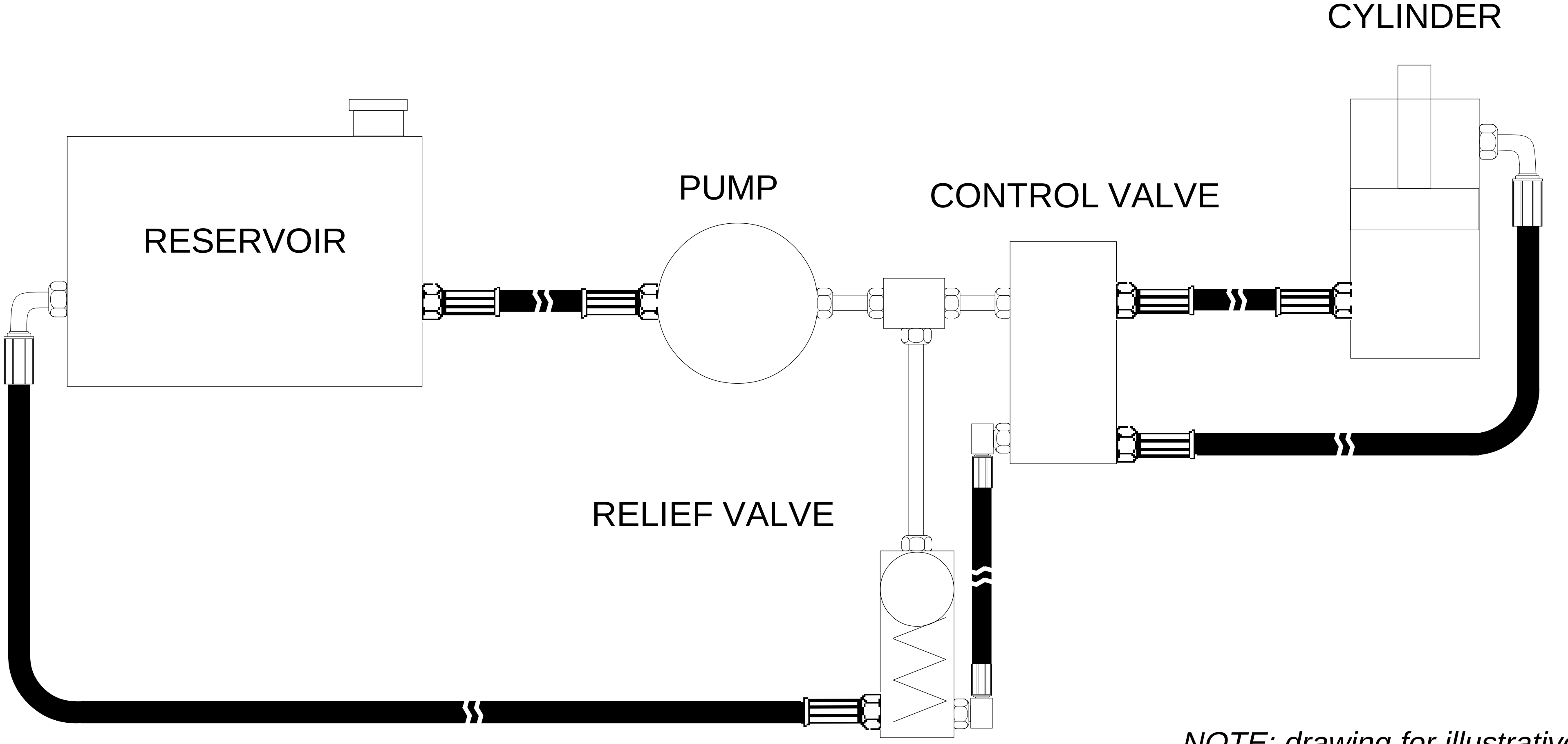
P = F_2 / A_2 = 1000 N / 100 cm² = 10 N / cm²

Force required on piston 1 (F_1):

P x A_1 = 10 N / cm² x 50 cm² = 500N

Assume a pump is connected and this movement takes 4 seconds:
speed = 0,8 m / 4 s = 0,2 m/s
volume (displaced) = 4l, hence pump displaces 4l in 4s and flow is 1l / s.

A simple hydraulic system



NOTE: drawing for illustrative purposes only.